DELHI PUBLIC SCHOOL, FARIDABAD

1st SEMESTER EXAMINATION 2016-2017

Maths

Class : X

Date : 21.09.16
MM : 90
Duration : 3 hrs.
P.S. : 3

General Instructions :-

1. All questions are compulsory.
2. The question paper consists of 31 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 11 questions of 4 marks each.
3. There is no overall choice.
4. Use of calculator is not permitted.

SECTION A

Q. 1 Explain why $7\times6\times5\times4+5$ is a composite number.

Q. 2 If A and B are acute angles of right ΔABC and $\sin A=\cos B$, then find the value of A+B.

Q. 3 Find mode, using empirical relation when it is given that mean and median are 10 and 9 respectively.

Q. 4 A ladder is placed against a wall such that its foot is at a distance of 5m from the wall and its top reaches a window $5\sqrt{3}$ m above the ground. Find the length of the ladder.

SECTION B

Q. 5 For what value of K will the following pair of linear equations has no solution

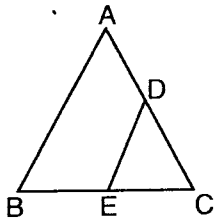
$Kx-3y+6=0$; $4x-6y+15=0$

Q. 6 If α and β are zeroes of the quadratic polynomial x^2-2x+3 ; find a polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Q. 7 If $\tan 2A = \cot(A-18°)$, where 2A is an acute angle, find the value of A.

Q. 8 Given $\tan A = \frac{4}{3}$, find cosecA.

Q. 9 In ΔABC, DE||AB. If AD=2x, DC=x+3, BE=2x−1 and CE=x, then find the value of x.



Q. 10 If $u_i = \frac{x_i - 25}{10}$, $\Sigma f_i u_i = 20$ and $\Sigma f_i = 100$; then find $\bar{x}$.

SECTION C

Q. 11 Prove that $\sqrt{3}$ is an irrational number.

Q. 12 Evaluate : $\tan^2 30° \sin 30° + \cos 60° \sin^2 90°. \tan^2 60° - 2\tan 45°.\cos^2 0° \sin 90°$

Q. 13 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m+1 for some integer m.

Q. 14 Solve for x and y graphically :

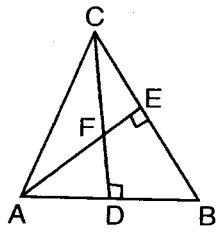
$2x+y-6=0$; $4x-2y-4=0$

Q. 15 In the given figure, altitudes AE and CD intersect at F.

Prove that :

a. $\Delta ADF \sim \Delta CEF$

b. $AD.EF = DF.CE$



Q. 16 Heights of students of class X are given in the following frequency distribution. Find the modal height.

Heights (in cm)	150-155	155-160	160-165	165-170	170-175
No. of Students	15	8	20	12	5

Q. 17 Diagonals AC and BD of a trapezium ABCD with AB||CD intersect each other at O. Prove that $\frac{OA}{OC}=\frac{OB}{OD}$.

Q. 18 Prove that :-

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$$

Q. 19 The weekly pocket money of a group of students is given below. Find the median for the data.

Pocket Money (in ₹)	0-40	40-80	80-120	120-160	160-200	200-240
No. of Students	5	7	15	10	5	8

Q. 20 If $x=\sec A+\sin A$ and $y=\sec A-\sin A$, prove that $\left(\frac{2}{x+y}\right)^2+\left(\frac{x-y}{2}\right)^2=1$

SECTION D

Q. 21 Atul, Ravi and Tarun go for a morning walk. They step off together and their steps measure 40cm, 42cm and 45cm respectively :

a. What is the minimum distance each should walk so that each can cover the distance in complete number of steps.

b. How is morning walk useful?

Q. 22 If the two zeroes of the polynomial $x^4+3x^3-20x^2-6x+36$ are $\sqrt{2}$ and $-\sqrt{2}$; find the other zeroes of the polynomial.

Q. 23 4 chairs and 3 tables cost ₹2100. 5 chairs and 2 tables cost ₹1750. Find the cost of one chair and one table separately.

Q. 24 Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

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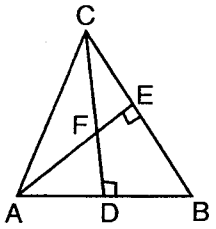

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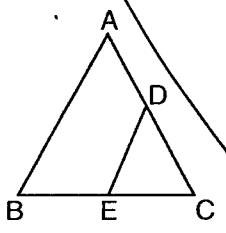

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Q. 12 Evaluate : $\tan^2 30° \sin 30° + \cos 60° \sin^2 90°. \tan^2 60° - 2\tan 45°.\cos^2 0° \sin 90°$

Q. 13 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m+1 for some integer m.

Q. 25 In annual day of a school, age-wise participation of students is shown in the following frequency distribution :

Age of Students (in years)	5-7	7-9	9-11	11-13	13-15	15-17	17-19
No. of Students	20	18	22	25	20	15	10

Draw a 'less than type' ogive for the above data and from it find the median age.

Q. 26 Prove that :

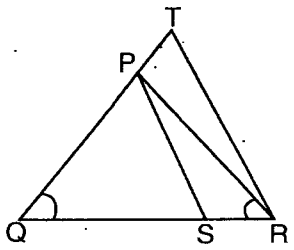
$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta.\operatorname{cosec}\theta$$

Q. 27 If α and β are zeroes of the polynomial $f(x)=x^2-6x+k$, find the value of k such that $\alpha^2+\beta^2=40$

Q. 28 Solve for x and y :

$$\frac{15}{x+y}-\frac{5}{x-y}=-2 \; ; \; \frac{10}{x+y}+\frac{2}{x-y}=4$$

Q. 29 In the given figure $\frac{QR}{QS}=\frac{QT}{PR}$ and $\angle PQS=\angle PRS$. Show that $\Delta PQS \sim \Delta TQR$.



Q. 30 The mean of the following distribution is 62.8.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	a	12	7	8

Find the missing frequency 'a'.

Q. 31 Prove that :

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$$

Ans | key Class X Maths SA I

Section A

Q1. $7 \times 6 \times 5 \times 4 + 5 = 5[7 \times 6 \times 4 + 1]$

$= 5m$; where $m = 7 \times 6 \times 4 + 1$] (1)

$\because$ 5 is a factor of given no.

$\therefore$ It is composite

Q2. $\sin A = \cos B$

$\sin A = \sin(90 - B)$

$\Rightarrow A = 90 - B$

$\Rightarrow A + B = 90°$] (1)

Q3. 3 Median = 3 | 1

Mode = 3 Median − 2 Mean

$= 3(4) - 2(9)$

$= 30 - 18$

$= 12$] (1)

Q4

$x \rightarrow$ Length of ladder] (1)

$x^2 = 25 + 75 = 100$

$x = 10\,m$

Sec - B

Q5. Condition for no solution (1/2)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ | $\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{K}{4} = \frac{-3}{-6}$ | $\frac{c_1}{c_2} = \frac{6}{15}$

$K = 2$ (1) | $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (1/2)

$\frac{b_1}{b_2} =$

(1/2)

Q6. $\alpha + \beta = 2$; $\alpha\beta = 3$

For new poly → Sum of zeroes $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{2}{3}$] (1)

→ Product of zeroes $= \frac{1}{\alpha\beta} = \frac{1}{3}$

New poly $= K\left[x^2 - \frac{2}{3}x + \frac{1}{3}\right]$ (1/2)

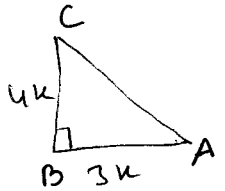
Q7. $\tan 2A = \tan[90 - A + 18]$] ①

$2A = 90 - A + 18$

$3A = 108$] ①

$A = 36$

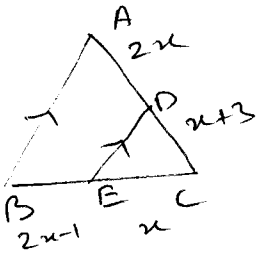
Q8



Hypotenuse $= 5k$ ①

$\text{Cosec}\, A = \frac{H}{P} = \frac{5k}{4k} = \frac{5}{4}$ ①

Q9.



By BPT

$\frac{2x}{x+3} = \frac{2x-1}{x}$] ①

$\Rightarrow 2x^2 = (x+3)(2x-1)$

$2x^2 = 2x^2 - x + 6x - 3$

$3 = 5x$

$\frac{3}{5} = x$] ①

Qw $\bar{u} = \frac{\bar{x} - 25}{10}$] ①

$\Rightarrow \frac{20}{10} = \frac{\bar{x} - 25}{10}$] ①

$\Rightarrow 21 = \bar{x}$

Sec-C

Q11. Let, us assume $\sqrt{3}$ is rational] ½

These exist coprimes integers p & q s.t $q \neq 0$ and

$\sqrt{3} = \frac{p}{q}$

$\Rightarrow q\sqrt{3} = p$

$\Rightarrow 3q^2 = p^2$ [SBS] ①

$\Rightarrow 3 \mid p^2$

$\Rightarrow 3 \mid p$] ①

$\Rightarrow p = 3m$ ② [for some integers m]

Fm ① & ②

$3q^2 = (3m)^2$

$\Rightarrow 3q^2 = 9m^2$

$\Rightarrow q^2 = 3m^2$

$\Rightarrow 3 \mid q^2$

$\Rightarrow 3 \mid q$] ①

$\Rightarrow$ 3 is a factor of both p & q

This contradicts the assumption that p & q are coprime. $\frac{1}{2}$

Hence our assumption is wrong

$\Rightarrow \sqrt{3}$ is irrational

Q12. $\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \cdot \tan^2 60^\circ - 2\tan 45^\circ \cos^2 0^\circ \sin 90^\circ$

$= \left(\frac{1}{\sqrt{3}}\right)^2 \cdot \frac{1}{2} + \frac{1}{2} \cdot (1)^2 \cdot (\sqrt{3})^2 - 2(1)(1)^2 \cdot (1)$ $1\frac{1}{2}$

$= \frac{1}{6} + \frac{3}{2} - 2$

$= \frac{1 + 9 - 12}{6}$ $1\frac{1}{2}$

$= -\frac{2}{6} = -\frac{1}{3}$

Q13. Any positive integer is of the form

$3q$, $3q+1$ OR $3q+2$

I: $n = 3q$

$n^2 = 9q^2$

$= 3(3q^2)$

$= 3m$; $m = 3q^2$ 1

II $n = 3q+1$

$n^2 = (3q+1)^2$

$= 9q^2 + 1 + 6q$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$; $m = 3q^2 + 2q$ 1

III: $n = 3q+2$

$n^2 = (3q+2)^2$

$= 9q^2 + 4 + 12q$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$; $m = 3q^2 + 4q + 1$ 1

Q14 $2x + y - 6 = 0 \Rightarrow y = -2x + 6$

x	0	1	2
y	6	4	2

$4x - 2y - 4 = 0 \Rightarrow y = 2x - 2$

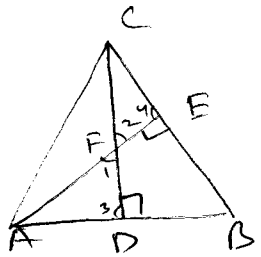
x	0	1	2
y	-2	0	2

1 – for plotting each line

$\frac{1}{2}$ → proper labelling of lines.

$\frac{1}{2}$ → solution (2, 2)

Q15



Given
TP
Fig] (1)

Pf. (1) In $\triangle ADF$ & $\triangle CEF$

$\angle 1 = \angle 2$ (Vo A)

$\angle 3 = \angle 4 = 90^\circ$ [AE & CD are altitudes]] (1½)

$\triangle ADF \sim \triangle CEF$ (AA)

(2) $\frac{AD}{CE} = \frac{DF}{EF}$ (Cpst)] (½)

$AD \cdot EF = CE \cdot DF$

Q16. Modal class = 160 – 165 (½)

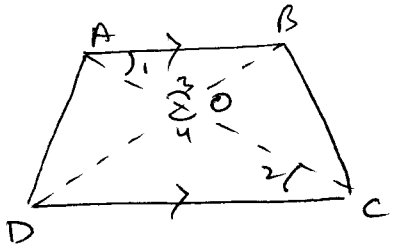
Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$] (1)

$= 160 + \frac{(20-8)}{2(20)-8-12} \times 5$

$= 160 + \frac{12^3 \times 5}{20_4}$] (1½)

$= 163$

Q17 Given
TP
Fig] (1)



Pf: In $\triangle AOB$ & $\triangle COD$

$\angle 1 = \angle 2$ [alt ∠s]

$\angle 3 = \angle 4$ Vo A

$\triangle AOB \sim \triangle COD$ (by AA)

$\frac{AO}{CO} = \frac{OB}{OD}$ (Cpst)] (2)

Q18 LHS = $\frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$ (1)

$= \frac{2 + 2\sin A}{\cos A(1+\sin A)}$ (1)

$= 2\sec A$ (1)

Q19

Pocket Money (in ₹)	No. of students (f)	cf
0 – 40	5	5
40 – 80	7	12
80 – 120	15	27
120 – 160	10	37
160 – 200	5	42
200 – 240	8	50

$n = 50$; $\frac{n}{2} = 25$

Median class $\longrightarrow$ 80 – 120

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 80 + \frac{25 - 12}{15} \times 40$$

$$= 114.67$$

(1 + 1/2)

Q20 $x + y = 2\sec A$

$x - y = 2\sin A$] (1 + 1/2)

$$LHS = \left(\frac{2}{2\sec A}\right)^2 + \left(\frac{2\sin A}{2}\right)^2$$

$$= \cos^2 A + \sin^2 A$$

$$= 1$$

(1 + 1/2)

Sec D

Q21. (a) Min dist = LCM (40, 42, 45) (1)

2 | 40, 42, 45	
3 | 20, 21, 45	
5 | 20, 7, 15	
4, 7, 3	

= 2520 cm

= 25.20 m

(2)

(b) (1)

Q22 $(x + \sqrt{2})$ & $(x - \sqrt{2})$ $\longrightarrow$ factors

$(x^2 - 2)$ $\longrightarrow$,,] (1)

$$x^2-2 \overline{)\, x^4+3x^3-2x^2-6x+36 \,(\, x^2+3x-18}$$

x^4 $\quad$ $-2x^2$

$\ominus$ $\quad$ $\oplus$

$3x^3 - 18x^2 - 6x$

$3x^3$ $\quad$ $-6x$

$\ominus$ $\quad$ $\oplus$

$-18x^2 + 36$

$-18x^2 + 36$

$\oplus$ $\quad$ $\ominus$

0

(2)

$x^2+3x-18 = (x+6)(x-3)$

other two zeroes are 6 & 3] (1)

Q23 chairs → x, table → y] (1½)

$$\begin{cases} 4x+3y = 2100 \times 2 \\ 5x+2y = 1750 \times 3 \end{cases}$$

$8x + 6y = 4200$

$15x + 6y = 5250$

$\ominus \quad \ominus \quad \ominus$

$-7x = -1050$

$x = 150$] (2)

$4(150) + 3y = 2100$

$\Rightarrow \quad 3y = 1500$

$y = 500$] (1)

Q24 Curves

TP

Eq] (1½)

Correct Pf (2½)

Q25

Age less than	Cf
7	20
9	38
11	60
13	85
15	105
17	120
19	130

(1)

Graph → (2 1/2)

Median → 11.4 (1/2)

Q26. $$\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \Big/ \frac{(\sin\theta-\cos\theta)}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \Big/ \frac{(\cos\theta-\sin\theta)}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$ (2)

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{\sin\theta\cos\theta} + 1$$ (2)

$$= 1 + \sec\theta\,\mathrm{cosec}\,\theta$$

Q27 $f(x) = x^2 - 6x + k$

If α & β are zeroes of $f(x)$

$\alpha + \beta = 6 \quad ; \quad \alpha\beta = k$] (1)

$\alpha^2 + \beta^2 = 40$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 40$] (1)

$\Rightarrow 36 - 2k = 40$

$\Rightarrow -2k = 4$

$\boxed{k = -2}$] (2)

Q28 $\frac{1}{x+y} = a \;, \quad \frac{1}{x-y} = b$

$(15a - 5b = -2) \times 2$

$(10a + 2b = 4) \times 3$] (1)

$30a - 10b = -4$

$30a + 6b = 12$

$-16b = -16$

$\boxed{b = 1}$

$15a - 5(1) = -2$

$15a = 3$

$a = \frac{1}{5}$ (2)

$x + y = 5$

$x - y = 1$

$2x = 6$

$\boxed{x = 3}$

$3 + y = 5$

$\boxed{y = 2}$] (2)

Q29. Given, TP, Fig] (1/2)

Pf: $\angle PQS = \angle PRS$ given

$PQ = PR$ [Isu Δ prop]

$\frac{QR}{QS} = \frac{QT}{PR}$ (Given)

$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ ($\because PQ = PR$)

$\angle Q = \angle Q$ (Common)

$\Delta PQS \sim \Delta TQR$ (SAS)] (2 1/2)

Q30

Class	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-20	5	10	-2	-10
20-40	8	30	-1	-8
40-60	a	50	0	0
60-80	12	70	1	12
80-100	7	90	2	14
100-120	8	110	3	24
	$\Sigma f_i = 40+a$			$\Sigma f_i u_i = 32$

(1)

$$\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$62.8 = 50 + \frac{(32)}{40+a} \times 20$$

(1½)

$$\Rightarrow \quad 12.8 = \frac{640}{40+a}$$

$$\Rightarrow \quad 12.8(40+a) = 640$$

$$\Rightarrow \quad 40+a = 50$$

$$\boxed{a = 10}$$

(1½)

Q31. LHS $= \dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$

$$= \frac{\{(\tan\theta + \sec\theta) - 1\}(\tan\theta - \sec\theta)}{\{\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$$

$$= \frac{\tan^2\theta - \sec^2\theta - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

(2)

$$= \frac{-1 - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= -\frac{(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$$

$$= \frac{1}{\sec\theta - \tan\theta}$$

(2)

(5) 4