DELHI PUBLIC SCHOOL, FARIDABAD

(B)

1st sem EXAMINATION, 2016-17

MARKING SCHEME

CLASS - XII SUBJECT - Maths DATE - 24/09 MAX MARKS - 100

Q. NO.		MARKS
1	No $\because$ $f(x_1)=f(x_2) \Rightarrow x_1 = \pm x_2$ It is not one-one	1
2	$\tan^{-1}(-1) = -\tan^{-1} 1 = -\pi/4$	1
3	$\lvert A\rvert=0$ gives the value of x = $\begin{vmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{vmatrix} = 0$ $x=0, 3$.	1
4	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b\sin\theta}{a\cos\theta} = -\frac{b}{a}\tan\theta$.	$\frac{1}{2}$
	$\frac{d^2y}{dx^2} = -\frac{b}{a}\sec^2\theta \cdot \frac{d\theta}{dx} = -\frac{b}{a}\sec^2\theta \times \frac{1}{a\cos\theta} = -\frac{b}{a^2}\sec^3\theta$	$\frac{1}{2}$
5	$\int \frac{x+\sin x}{1+\cos x}\,dx = \frac{1}{2}\int \underset{u}{x} \cdot \underset{v}{\sec^2\frac{x}{2}} + \int \tan\frac{x}{2}\,dx$	$\frac{1}{2}$
	$= \frac{1}{2}\cdot\left(x \frac{\tan\frac{x}{2}}{\frac{1}{2}} - \int \frac{\tan\frac{x}{2}\,dx}{\frac{1}{2}}\right) + \int \tan\frac{x}{2}\,dx$	1
	$= x\tan\frac{x}{2} + c$	$\frac{1}{2}$
6	$y=\pi r^2$ where $r = 5$ cm $r+\Delta r = 5.1$ cm	$\frac{1}{2}$
	$\Rightarrow \Delta r = 0.1$ cm $\therefore \frac{dy}{dr} = \frac{\Delta y}{\Delta r}$,	$\frac{1}{2}$
	$\Rightarrow \Delta y = 2\pi r \cdot \Delta r = 2\pi \cdot 5 \cdot (0.1) = \pi$ cm^2	1
7	At x-axis $y=0$, $\Rightarrow x=7$ pt is $(7,0)$	$\frac{1}{2}$
	Differentiate, to get slope of normal at $(7,0)$ as -20	1
	$\therefore y-0 = -20(x-7) \Rightarrow 20x+y = 140$	$\frac{1}{2}$

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MARKING SCHEME

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Q. NO.		MARKS
8	$f'(x) = 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}$	1
	Clearly $f'(x) > 0 \ \forall \ x \in R$ (Show the working)	1
9	$y + x\frac{dy}{dx} + y \cdot 1 = x(\sin x + \log x)$.	
	$\frac{dy}{dx} + \frac{2y}{x} = \sin x + \log x$	$\frac{1}{2}$
	$I.F = e^{\int \frac{2}{x}} = e^{2\log x} = x^2$	$\frac{1}{2}$
	$y \cdot x^2 = \int x^2 \cdot (\sin x + \log x) \cdot dx$ Solve it	1
10	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t}{\frac{a\cos^2 t}{\sin t}} = \tan t$	$1\frac{1}{2}$
	$\Rightarrow \left.\frac{dy}{dx}\right)_{x=\pi/4} = 1$	$\frac{1}{2}$
11	$\int_{-\pi/2}^{\pi/2} \log_e\left(\frac{2+x}{2-x}\right) dx = 0$	
	(Show $f(-x) = -f(x)$)	1
	$\therefore$ Applying property	1
12	$f(x)$ is diff in $(-1,1)$, cont in $[-1,1]$	
	$f(-1) = f(1)$	1
	Now $f'(c) = 0 \ \forall \ c \in (-1,1)$ gives $c = 0$	1
13	$\cos^{-1}\left(\frac{xy}{6} - \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right) = \theta$.	1
	$\Rightarrow \frac{xy}{6} - \cos\theta = \sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}$	1
	Squaring and solving	2

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1st semester EXAMINATION, 2016-17 (3)

MARKING SCHEME

CLASS - XI SUBJECT - XI DATE - 24/09 MAX MARKS - 100

Q. NO.		MARKS
14(b)	$R_1 \to R_1 + R_2 + R_3$	1
	$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ Now $\left.\begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - 1 \end{matrix}\right\}$	1
	$(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$ Expand along first row and solve	2
15	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ and $x_1 x_2 = 1$	2
	Also range of f is $y \in \left[-\frac{1}{2}, \frac{1}{2}\right] \neq$ co domain of f (Calculate the range)	2
	OR	
	Reflexive + Symmetric + Transitive	1+1+2
16	$x = \tan\theta \cdot dx = \sec^2\theta\, d\theta$	1
	$\Rightarrow 2\int \underset{u}{\theta} \cdot \underset{v}{\sec^2\theta}\, d\theta$	1
	Solving the integral	2
	OR	
	$\int \underset{v}{\frac{x\cos x}{(x\sin x+\cos x)^2}} \cdot \underset{u}{x\sec x}\, dx$	1
	Solving the integral	
	Answer: $\frac{-x}{\cos x(x\sin x+\cos x)} + \tan x + c = \frac{\sin x - x\cos x}{x\sin x+\cos x} + c$	3

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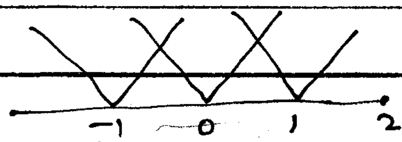
1st sem EXAMINATION, 2016-17 (4)

MARKING SCHEME

CLASS - XII SUBJECT - Maths DATE - 24/09 MAX MARKS - 100

Q. NO.		MARKS
(17)		
	$= \int_{-1}^{0} (x+1) - x - (x-1)\cdot dx + \int_{0}^{1} (x+1) + x - (x-1) dx$	(1) + (1)
	$+ \int_{1}^{2} x+1+x+x-1\cdot dx$	
	= Getting the result	(2)
	(OR)	
	$x^2=t$ $\quad \frac{t}{(t+1)(t+4)} = \frac{A}{t+1} + \frac{B}{t+4}$ $\therefore A = \_\_\_$	(1)
	$B = \_\_\_$	(1)
	$\Rightarrow \int \frac{A}{x^2+1} + \frac{B}{x^2+2^2}\cdot dx = A\tan^{-1}x + \frac{B}{2}\tan^{-1}\frac{x}{2} + C$	(2)
(18)	$\frac{dx}{dy} = \frac{\sin^{-1}y}{\sqrt{1-y^2}} - \frac{x}{\sqrt{1-y^2}}$	
	$I.F = e^{\int \frac{dy}{\sqrt{1-y^2}}} = e^{\sin^{-1}y}$	(1)
	$x\cdot e^{\sin^{-1}y} = \int e^{\sin^{-1}y}\cdot \frac{\sin^{-1}y}{\sqrt{1-y^2}} dy + c$	(1)
	$\sin^{-1}y = t$	
	$\Rightarrow x\cdot e^{\sin^{-1}y} = e^{\sin^{-1}y}(\sin^{-1}y - 1) + c$	
	Put $x=0, y=0$, to get c	(1)
	$\therefore x = \sin^{-1}y - 1 + e^{-\sin^{-1}y}$	(1)
(19)	$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$	(1)
	Solve to get $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$	(3)

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MARKING SCHEME

CLASS - XII SUBJECT - Maths DATE - 24/09 MAX MARKS - 100

Q. NO.		MARKS
20	$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = f(\frac{\pi}{4}) \Rightarrow f(\frac{\pi}{4}) = \frac{1}{2}$	
	or	4
	$\lim_{x\to 2} \frac{2^{x+2}-16}{4^x-16} = \cancel{tan}\, f(2) \Rightarrow k = \frac{1}{2}$	
21	$y = x^x \Rightarrow \frac{dy}{dx} = x^x(1+\log_e x)$	1
	$\frac{d^2y}{dx^2} = x^x \cdot \left(\frac{1}{x}\right) + (1+\log_e x) \cdot d(x^x)$	1
	$\Rightarrow x^x \cdot \frac{1}{x} + x^x \cdot (1+\log x)^2$	2
	$= x^x \cdot \left((1+\log_e x)^2 + \frac{1}{x}\right)$	
22	$\frac{dy}{dx} - \frac{y}{x} = \left(\frac{x-1}{x}\right) e^x$	
	$I.F = e^{-\int\frac{1}{x}} = e^{-\log_e x} = e^{\log_e x^{-1}} = \frac{1}{x}$	1
	$y \cdot \frac{1}{x} = \int \frac{x-1}{x} \cdot e^x \cdot \frac{1}{x} = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) \cdot dx + C$ Solving to get.	1
	$\Rightarrow y/x = \frac{e^x}{x} + C$	2
	(or)	
	Show $f(\lambda x, \lambda y) = \lambda^0 f(x,y)$	1
	Put $y = zx \Rightarrow \frac{dy}{dx} = z + x\frac{dz}{dx}$	1
	$\Rightarrow \frac{z\cos z + 1}{\cos z} - z = x\frac{dz}{dx}$	1
	$\Rightarrow \frac{1}{\cos z} = x\frac{dz}{dx}$	
	$\Rightarrow \frac{dx}{x} = \cos z\, dz$	1

$\log_e x = \sin z + C$ Put z. solving

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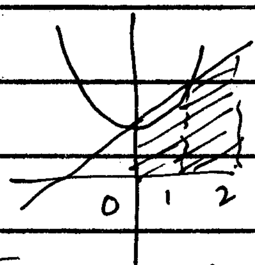
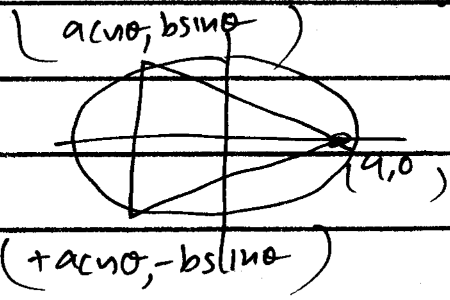
MARKING SCHEME

CLASS - XII SUBJECT - Maths DATE - 24/09 MAX MARKS - 10

Q. NO		MARKS
23	$\int \frac{x^2+1 \cdot x^2+2}{x^2+3 \cdot x^2+4}$ Put $x^2=t$	½
	$\cancel{x^2+1\cdot x} \Rightarrow \frac{t+1 \cdot t+2}{t+3 \cdot t+4} = 1 + \frac{A}{t+3} + \frac{B}{t+4}$	½
	$A = \checkmark \quad B = \checkmark$	1
	$\therefore \int 1 + \frac{A}{x^2+\sqrt{3}^2} + \frac{B}{x^2+2^2} \cdot dx = x + \frac{A}{\sqrt{3}} \tan^{-1}\frac{x}{\sqrt{3}} + \frac{B}{2}\tan^{-1}\frac{x}{2}$	2
24	 $A = \int_0^1 x^2+1 \cdot dx + \int_1^2 x+1 \cdot dx$ $= \frac{23}{6}$ unit^2.	2+2+2
25	$\sqrt{\tan x} = t$ $\tan x = t^2 \Rightarrow \sec^2 x\,dx = 2t\,dt$	1
	$\therefore \int \frac{2t^2}{t^4+1} \cdot dt = \int \frac{t^2+1}{t^4-1} \cancel{dt}$ $\int \frac{t^2-1}{t^4+1}\,dt$ } Solving both integrals	2+2
	Put $t = \sqrt{\tan x}$ to get the result.	1
26	 $A = \frac{1}{2} \cdot 2b\sin\theta \cdot (a + a\cos\theta)$	1+1
	$\frac{dA}{d\theta} = 0$ gives $\theta = \pi/3$	1
	$\left.\frac{d^2A}{d\theta^2}\right\} = -ve$	2

$A_{max} = ab \cdot \frac{\sqrt{3}}{2} \cdot \left(1+\frac{1}{2}\right)$

$= \frac{3\sqrt{3}ab}{4}$ unit^2 (1)

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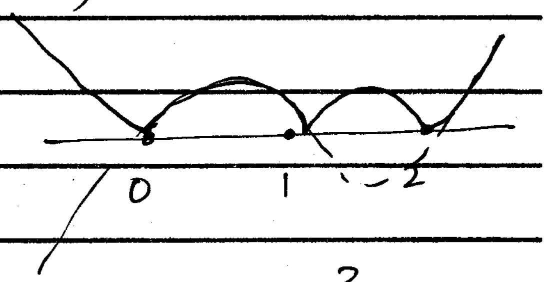
MARKING SCHEME

CLASS - XII SUBJECT - Maths DATE - 24/09 MAX MARKS - 100

Q. NO.		MARKS
27	$x+y+z=70$ $2x+3y+4z=210$ $5y+4z=230$	1
	$A^{-1} = \frac{-1}{6}\begin{bmatrix} -8 & 1 & 1 \\ -8 & 4 & -2 \\ 10 & -5 & 1 \end{bmatrix}$	3
	Solving $x=20$, $y=30$, $z=20$ + Value base	1
28	(a) $\lvert x(x-1)(x-2) \rvert$ 	
	$\int_{-1}^{2} \lvert x^3-x \rvert \cdot dx = \int_{-1}^{0} x-x^3 \, dx + \int_{0}^{1} x^3-x \, dx + \int_{1}^{2} x-x^3 \cdot dx$	1+1+1
	+ $\int$ on solving.	
	Adding to get answer	2
(b)	$I = \int_0^{\pi/2} f(\sin 2x) \sin x \, dx$ (Prop IV).	1
	and add $2I = \int_0^{\pi/2} f(\sin 2x)(\sin x + \cos x) \, dx$	
	$\Rightarrow 2I = \sqrt{2}\int_0^{\pi/2} f(\sin 2x) \cdot \sin\left(x+\frac{\pi}{4}\right) \cdot dx$ (Prop IV)	1
	and solving to get RHS.	1

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MARKING SCHEME

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Q. NO.		MARKS
29	$f(1) = \frac{1+1}{2} = 1$	
	$f(2) = \frac{2}{2} = 1$	
	$\therefore f(1) = f(2) = 1$	2
	$\Rightarrow f$ is many-one	
	let $n \in N$.	
	If n is odd, $2n-1$ is also odd	2
	$\therefore f(2n-1) = \frac{2n-1+1}{2} = n$	
	If n is even, $2n$ is also even	2
	$\therefore f(2n) = \frac{2n}{2} = n$.	
	$\therefore f$ is onto	

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